wizEXPERT 2024

THURSDAY
MARCH 21ST 2024

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WERELDWIJDE WISKUNDE WEDSTRIJD

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GOOD LUCK AND MOST OF ALL HAVE FUN !

calculators are not allowed

you may use 75 minutes

only a pencil, an eraser and scribbling paper are allowed

results and prizes will arrive at school at the end of May

answers will be posted on the website about March 29th

wizEXPERT
WO students
HBO students

1. Eight equal pentagons with different imprints are put together.
One closed self-intersecting loop is formed.
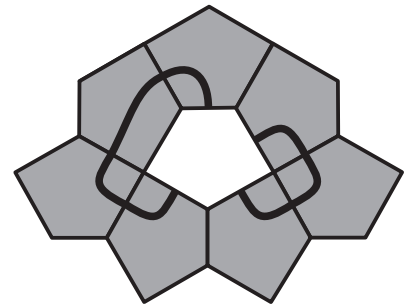
Which pentagon is missing in the middle?

A. **B.** **C.** **D.** **E.**

2. Which of the following numbers is two less than a multiple of ten, two more than a square and two times a prime number?

A. 6 **B.** 18 **C.** 38 **D.** 58 **E.** 78

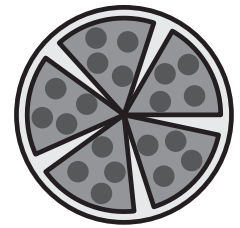**3.** A young kangaroo cut a pizza into six equal pieces.
After eating one piece, he arranged the remaining pieces so that the gaps between them were equal.

What size is the angle of each gap?

A. 5° **B.** 8° **C.** 9° **D.** 10° **E.** 12°

4. *Thomas* has the unusual habit of drawing the coordinate system with the positive coordinate axes pointing left and down.

What does the graph of the line $y = x + 1$ look like in *Thomas's* coordinate system?

A. **B .** **C.** **D.** **E.**

5. *Simon* has made an unusual die. The probabilities of rolling a 2, a 3, a 4 and a 5 are still $\frac{1}{6}$, but the probability of rolling a 6 is now twice that of rolling a 1.
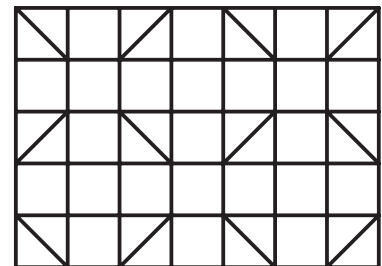
What is the probability of rolling a 6?

A. $\frac{1}{6}$ **B.** $\frac{7}{36}$ **C.** $\frac{2}{9}$ **D.** $\frac{1}{4}$ **E.** $\frac{5}{18}$

6. Which of the following expressions below is equal to $16^{15} + 16^{15} + 16^{15} + 16^{15}$?

A. 4^{31} **B.** 16^{19} **C.** 4^{60} **D.** 16^{60} **E.** 4^{122}

7. *Beaver* wants to colour the tiling in the figure alongside.
Tiles adjacent to each other, even those sharing a single vertex, are painted different colours.

At least how many colours does *Beaver* need?

A. 3 **B.** 4 **C.** 5 **D.** 6 **E.** 7

8. There are 6 glasses upright on the table.
With each move, exactly 4 glasses are turned over.

What is the smallest number of moves to get all the glasses upside down?

A. 2 **B.** 3 **C.** 4 **D.** 5 **E.** 7

9. *Leon* makes a sequence of numbers. He starts with 1 and multiplies it by either 6 or 10.
Then he starts multiplying the answer again by either 6 or 10.
He continues this procedure many times.

Which of the following numbers can definitely **not** be in his sequence?

A. $2^{50}\,5^{50}$ **B.** $2^{80}\,3^{40}\,5^{40}$ **C.** $2^{90}\,3^{20}\,5^{80}$ **D.** $2^{100}\,3^{20}\,5^{80}$ **E.** $2^{110}\,3^{80}\,5^{30}$

10. A black and a white trail cross a park as shown in the picture.
Each trail divides the park into two regions of equal area.
Below are five statements about the areas *A*, *B* and *C*.
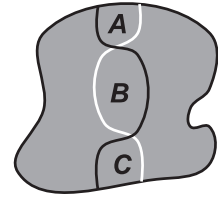

Which one is definitely true?

A. $A = C$ **B.** $B = \frac{1}{2}(A + C)$ **C.** $B = \frac{3}{5}(A + C)$ **D.** $B = \frac{2}{3}(A + C)$ **E.** $B = A + C$

11. Five statements are made about a positive integer *n*.
Exactly one of these statements is true.
The other four are not true.

Which statement about *n* is true?

A. *n* is divisible by 3   **B.** *n* is divisible by 6   **C.** *n* is odd
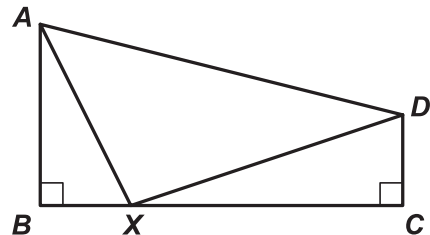**D.** *n* = 2   **E.** *n* is prime

12. A triangular pyramid *ABCD* has sides of length 5, 6, 7, 8, 9 en 10.
The points *M*, *N*, *P*, *Q*, *R* and *S* are the midpoints of the edges, see the figure.
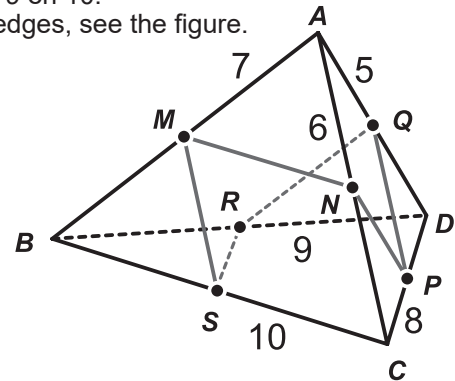
What is the length of the path *MNPQRSM*?

A. $20\frac{1}{2}$ **B.** 21   **C.** $21\frac{1}{2}$ **D.** 22   **E.** $22\frac{1}{2}$

13. A quadrilateral *ABCD* has two right angles at *B* and *C*.
AB = 4, *BC* = 8 en *CD* = 2 .
X is a point on *BC*.

What is the minimum value of *AX* + *DX*?

**A.** 10   **B.** 12   **C.** $9\sqrt{2}$ **D.** 13   **E.** any other answer

14. *Iris* has a number of all-black and all-white unit cubes.
She wants to build a larger cube with 27 of these unit cubes.
The surface of this larger cube is exactly half white and half black.

What is the smallest number of black cubes *Iris* needs?

A. 11   **B.** 12   **C.** 13   **D.** 14   **E.** any other answer
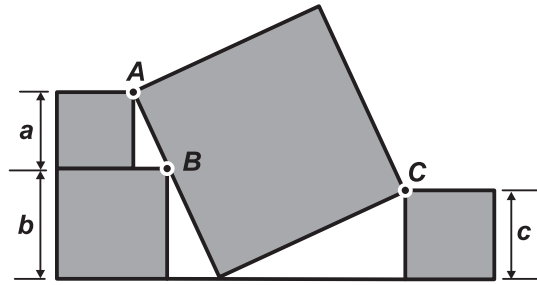
15. In a 6 by 6 square, a diagonal, a semicircle and a quarter circle are drawn.
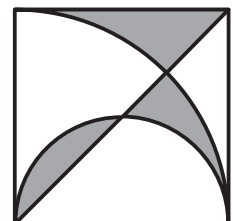
What is the area of the shaded region?

A. 9   **B.** 3π **C.** $6\pi - 9$ **D.** $\frac{10\pi}{3}$ **E.** 12

16. The figure shows four squares. The smaller ones have sides *a*, *b*, and *c*. The vertices *A* and *C* of two of the smaller squares are the endpoints of the diagonal of the large square. Vertex *B* of the third small square is on a side of the large square.

What is the side length of the large square equal to?

A. $\frac{1}{2}(a + b + c)$

B. $\sqrt{a^2 + b^2 + c^2}$

C. $\sqrt{(a + b)^2 + c^2}$

D. $\sqrt{(b - a)^2 + c^2}$

E. $\sqrt{a^2 + ab + b^2 + c^2}$

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17. Given are two positive numbers *p* and *q*, with $p < q$.

Which of the following expressions is the largest?

A. $\frac{p + 3q}{4}$ **B.** $\frac{p + 2q}{3}$ **C.** $\frac{p + q}{2}$ **D.** $\frac{2p + q}{3}$ **E.** $\frac{3p + q}{4}$

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18. How many three-digit numbers contain at least one of the digits 1, 2 and 3?

A. 27 **B.** 147 **C.** 441 **D.** 551 **E.** 606

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19. *N* is a four-digit number, $N = pqrs$.
If you put a decimal point between the *q* and the *r*, then the resulting number *pq,rs* is the average of the two-digit numbers *pq* and *rs* .

What is the sum of the digits of *N*?

A. 14 **B.** 18 **C.** 21 **D.** 25 **E.** 27

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20. Two candles of equal length are lit simultaneously.
Both candles burn at their own constant rate.
One of the candles will burn down in 4 hours and the other in 5 hours.

After how many hours is one candle three times the length of the other?

A. 3 **B.** $\frac{63}{20}$ **C.** $\frac{47}{14}$ **D.** $\frac{40}{11}$ **E.** $\frac{45}{12}$

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21. Six cards have one number on each side. The pairs of numbers on the cards are (5,12), (3,11), (0,16), (7, 8), (4,14) and (9,10). A card is placed on each of the empty spaces below.

$$\square + \square + \square - \square - \square - \square = \,?$$
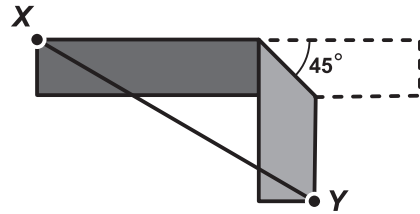
What is the smallest possible result?

A. −27 **B.** −26 **C.** −25 **D.** −24 **E.** −23

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22. *Kangaroo* solves the equation $ax^2 + bx + c = 0$ and *Beaver* solves the equation $bx^2 + ax + c = 0$, where *a*, *b* and *c* are pairwise distinct non-zero integers.
It turns out that the equations have a common solution.

Which of the following statements is then definitely true?

A. $a + b = c$
B. The equation $ax^2 + bx + c = 0$ has exactly one real solution.
C. $a > 0$
D. $b < 0$
E. $a + b + c = 0$

23. A strip of paper that is 12 cm long and 2 cm wide is folded over 45°.

What is, in cm, the smallest possible length of XY?

A. $6 + \sqrt{2}$ **B.** 8   **C.** $6\sqrt{2}$ **D.** $7\sqrt{2}$ **E.** 10

24. *Kyra* has several unbiased 12-sided dice, each with the numbers 1 to 12 on the faces.
If *Kyra* rolls all the dice at once, the probability of rolling a 12 exactly once is equal to the probability of rolling no 12 at all.

How many dice does *Kyra* have?

A. 8   **B.** 9   **C.** 10   **D.** 11   **E.** 12

25. A polynomial p satisfy the relation $p(x + 1) = x^2 - x + 2p(6)$ for every real x.

What is the sum of the coefficients of p?

A. −40   **B.** −6   **C.** 6   **D.** 12   **E.** 40

26. If $2^x = 3$, $2^y = 7$ and $6^z = 7$, which of the following is true for z?

A. $z = \frac{y}{1 + x}$ **B.** $z = \frac{x}{y} + 1$ **C.** $z = \frac{y}{x} - 1$ **D.** $z = \frac{x}{y - 1}$ **E.** $z = y - \frac{1}{x}$

27. A strip of paper consists of eight squares. Initially each square contains the number 0.
Leonie chooses four consecutive squares and adds 1 to the numbers in those squares.
Below you can see the strip after *Leonie* has done this several times.
Unfortunately, ink has fallen on some squares.
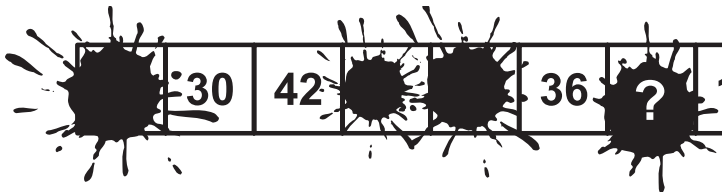
What number is written on the square with the question mark?

A. 24   **B.** 30   **C.** 36   **D.** 48   **E.** any other answer

28. A function $f: \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(20 - x) = f(22 + x)$ for all real x.
Given that f has exactly two roots.

What is the sum of these two roots?

A. −1   **B.** 20   **C.** 21   **D.** 22   **E.** any other answer

29. Twelve points are equally spaced on a circle.
We draw triangles with three of these points as vertices.

How many of these triangles have an angle of 45°?

A. 48   **B.** 60   **C.** 72   **D.** 84   **E.** 96

30. N is a four-digit number, $N = abcd$.
It satisfies the equation $N = a^a + b^b + c^c + d^d$.

What is the value of digit a?

A. 2   **B.** 3   **C.** 4   **D.** 5   **E.** 6