wizEXPERT 2023

WereldWijde WiskundeWedstrijd

WWW.W4KANGOEROE.NL

W4Kangoeroe

THURSDAY MARCH 16TH 2023

Good luck and most of all have fun !

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calculators are not allowed

only a pencil, an eraser and scribbling paper are allowed

answers will be posted on the website about March 29th

you may use 75 minutes
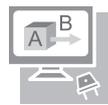
results and prizes will arrive at school at the end of May

wizEXPERT
WO students
HBO students

1. What is the value of the fraction $\frac{7777^2}{5555 \cdot 2222}$ when you simplify it?

    A. 1   **B.** $\frac{7}{10}$ **C.** $\frac{49}{10}$ **D.** $\frac{77}{110}$ **E.** 49

2. *Julia* rolls five dice and scores 19 points.

What is the maximum number of sixes she could have thrown?

    A. 0   **B.** 1   **C.** 2   **D.** 3   **E.** 4

3. A cylindrical tin can is 15 cm high.
The perimeter of the bottom is 30 cm.
An ant walks from point A on the bottom to point B on the top edge.
The animal walks either vertically or horizontally.
The route is shown in the figure to the right.
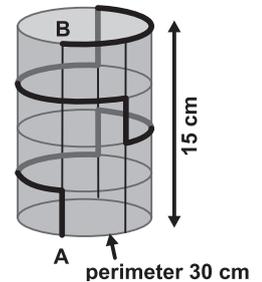
How many cm does the ant walk?

    A. 45   **B.** 55   **C.** 60   **D.** 65   **E.** 75

4. *Hamza* wants to paint the three horizontal strips of this flag.
He has four colours of paint. Each strip gets one colour.
Each colour may be used more than once.
Stripes next to each other should not have the same colour.
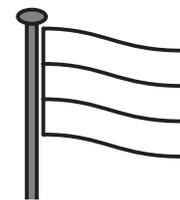
In how many different ways can *Hamza* paint the flag?

    A. 24   **B.** 27   **C.** 32   **D.** 36   **E.** 64

5. A positive integer n is called 2-*primy* if it has exactly three different divisors: 1, 2 and n itself.

How many different 2-*primy* numbers are there?

    A. 0   **B.** 1   **C.** 2   **D.** 3   **E.** 4

6. How many different pairs of positive integers x, y satisfy the equation $x + 2y = 2^{10}$?

    A. 0   **B.** $2^9 - 1$ **C.** 2^9 **D.** $2^9 + 1$ **E.** $2^9 + 2$

7. Two equilateral triangles are put on each other.
The overlapping part is a hexagon with opposite sides parallel.
The lengths of four sides of the hexagon are given in the figure.
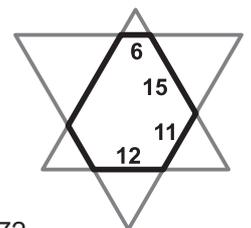
What is the perimeter of the hexagon?

    A. 64   **B.** 66   **C.** 68   **D.** 70   **E.** 72

8. A square with area 84 is divided into four squares.
The upper left square is coloured black.
The lower right square is also divided into four squares again,
of which the upper left square is coloured black, etc.
This process is repeated infinitely many times.

What is the area of the black coloured area?

    A. 24   **B.** 28   **C.** 31   **D.** 35   **E.** 42

9. In the boxes below, all numbers from 1 to 9 must be filled in.
The sum of three consecutive numbers must always be a multiple of 3.
The numbers 7 and 9 have already been placed.

	7		9					

In how many ways can the remaining boxes be filled?

    A. 9   **B.** 12   **C.** 15   **D.** 18   **E.** 24

10. With which digit does the product $(5^5 + 1)(5^{10} + 1)(5^{15} + 1)$ end?

A. 0   **B.** 1   **C.** 3   **D.** 5   **E.** 6

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11. A triangular pyramid has six edges.
The lengths of the edges are integers.
The lengths of four of these edges are given in the figure alongside.
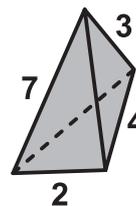
What is the sum of the lengths of the remaining two edges?

A. 9   **B.** 10   **C.** 11   **D.** 12   **E.** 13

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12. For any positive integer n, $n!$ is the product of the integers 1 up to n.
For example $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$. Of the number g, we know that $g! = 6! \cdot 7!$.

What is the sum of the digits of the number g?

A. 1   **B.** 2   **C.** 4   **D.** 8   **E.** 9

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13. A point A lies for every possible value of a on the graph of $y = x^3 + 3x^2 + ax + 2a + 4$.

What is the sum of the coordinates of point A?

A. 2   **B.** 4   **C.** 6   **D.** 8   **E.** 10

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14. Given are five numbers a_1, a_2, a_3, a_4 and a_5.
The sum of these numbers is S.
They are related by the formula $a_k = k + S$ for $k = 1, 2, 3, 4, 5$.

What is the value of S?

A. -15 **B.** $-\frac{15}{4}$ **C.** $\frac{15}{4}$ **D.** 15   **E.** you can not know

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15. How many pairs of integers m, n satisfy the inequality
$$|2m - 2023| + |2n - m| \le 1?$$

A. 0   **B.** 1   **C.** 2   **D.** 3   **E.** 4

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16. There are 23 animals in a row.
Each animal is either a kangaroo or a beaver.
Each animal has at least one kangaroo as a neighbor.

What is the maximum number of beavers in this row?

A. 7   **B.** 8   **C.** 10   **D.** 11   **E.** 12

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17. We can write 5^{5^6} as n^n for a certain number n.

Which number is n?

A. 11   **B.** 30   **C.** 5^5 **D.** 5^6 **E.** 5^{30}

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18. *Leon* has drawn some curves on the net of a rectangular beam.

Which net can he fold so that there is one closed curve on the beam?

A.   **B.**   **C.**   **D.**   **E.**
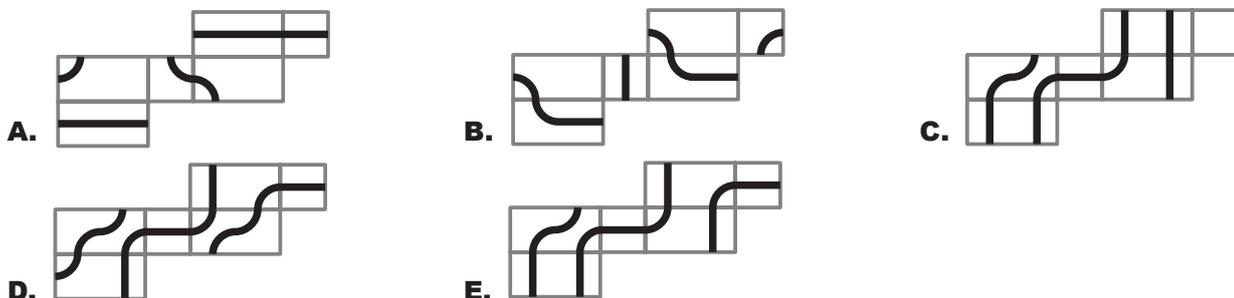
19. A pentagon is divided into smaller parts as shown on the right. The number inside the traingles indicate their area.
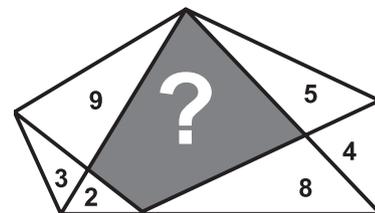
What is the area of the grey part with the question mark?

A. 15 **B.** $\frac{31}{2}$ **C.** 16 **D.** 17 **E.** you can not know

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20. How many numbers divide $2^{20}3^{23}$ but do not divide $2^{10}3^{20}$?

A. 13 **B.** 30 **C.** 273 **D.** 460 **E.** you can not know

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21. Two functions f and g on $\mathbb{R}$ satisfy the system of equations $f(x) + 2g(1-x) = x^2$ and $f(1-x) - g(x) = x^2$.

Which function is f ?

A. $f(x) = x^2 - \frac{4}{3}x + \frac{2}{3}$ **B.** $f(x) = x^2 + \frac{4}{3}x + \frac{2}{3}$ **C.** $f(x) = -x^2 + \frac{4}{3}x + \frac{2}{3}$

D. $f(x) = x^2 - 4x + 5$ **E.** there is no such function

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22. In a climbing tournament, 13 climbers compete in three categories.
The score of each competitor is the product of their rankings in the three categories.
So if someone finishes 4th, 3rd and 6th the score is $4 \cdot 3 \cdot 6 = 72$.
Hannah becomes 1st in two categories.

At most, how many climbers can have a lower score than *Hannah*?

A. 1 **B.** 2 **C.** 3 **D.** 4 **E.** 5

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23. Consecutive numbers are written down in a spiral as below, starting with 1.
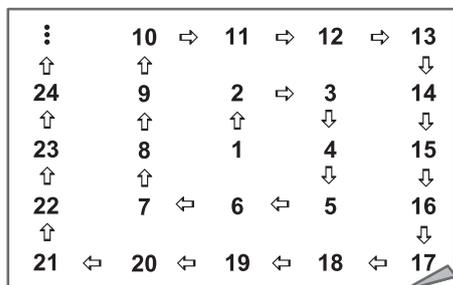
If we continue the pattern, in what shape will the numbers 625, 626 and 627 appear?

A.
627
⇧
626
⇧
625

B.
626 ⇨ 627
⇧
625

C. 625 ⇨ 626 ⇨ 627

D.
625 ⇨ 626
⇩
627

E.
625
⇩
626
⇩
627

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24. The large square is divided into four smaller squares.
The circle touches the right side of the large square at its midpoint.
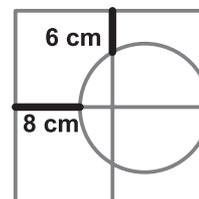
How many cm is the side lenght of the large square?

A. 18 **B.** 20 **C.** 24 **D.** 28 **E.** 30

25. In the regular tetrahedron below, one face is coloured grey.
The tetrahedron is placed with the grey face on the board to the right on the triangle with START.
The tetrahedron is rolled over the board by rotating the tetrahedron over its edges.
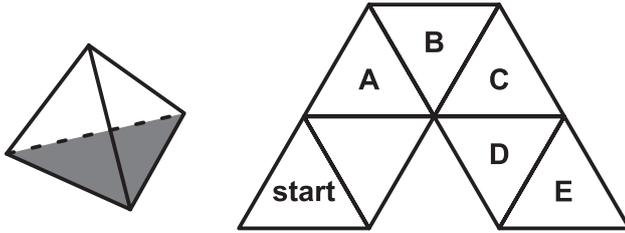
On which triangle will the tetrahedron stand for the first time again on its grey face?

A. A　　　　**B.** B　　　　**C.** C　　　　**D.** D　　　　**E.** E

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26. Because of an inkblot, part of the 5th-degree polynomial cannot be seen.

It is known that all the roots of this polynomial are integers.

What is the highest power of $x - 1$ that divides the polynomial?

A. $(x - 1)^1$　　　**B.** $(x - 1)^2$　　　**C.** $(x - 1)^3$　　　**D.** $(x - 1)^4$　　　**E.** $(x - 1)^5$

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27. What is the greatest common divisor of all numbers of the form $n^3(n + 1)^3(n + 2)^3(n + 3)^3(n + 4)^3$, where n is a positive integer?

A. $2^9 3^3$　　　**B.** $2^6 3^3 5^3$　　　**C.** $2^8 3^2 5^3$　　　**D.** $2^8 3^3 5^3$　　　**E.** $2^9 3^3 5^3$

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28. The numbers from 1 to 11 must be written in the hexagons.
Around each black point the sum of the numbers should be the same.
Three numbers are already written.
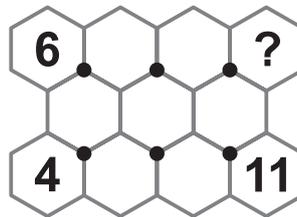
Which number will be written in the hexagon with the question mark?

A. 1　　　　**B.** 3　　　　**C.** 5　　　　**D.** 7　　　　**E.** 9

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29. Two identical cylinders contain the same amount of water.
One of the cylinders is standing straight, the other is leaning against it.
The bottom of the right cylinder is just barely completely covered with water.
The water level is the same in both cylinders, as shown here.
The bottom of both cylinders has an area of 3π m².
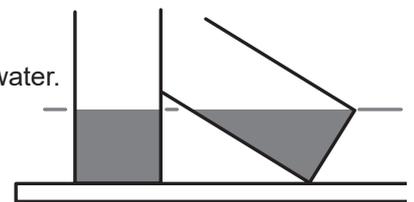
How many m³ of water does each of the cylinders contain?

A. $\frac{3\pi}{4}$　　　**B.** $3\sqrt{3}\pi$　　　**C.** 6π　　　**D.** 9π　　　**E.** you can not know

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30. The product of six consecutive numbers is a 12-digit number of the form

$$abbcddcddabb$$

where the digits a, b, c and d are themselves consecutive in some order.

Which digit is d?

A. 1　　　　**B.** 2　　　　**C.** 3　　　　**D.** 4　　　　**E.** 5